Introduction to Abstract Algebra

with Applications to Social Systems

Women in Mathematics
SWIM
Summer Workshop in Mathematics
FOR HIGH SCHOOL STUDENTS
Princeton

Course II
Lecture
Notes
7 of 7

Princeton SWIM 2010
Instructor: Taniecea A. Arceneaux
Teaching Assistants: Sarah Trebat-Leder and Amy Zhou

Southern Women Data

Davis, Gardner, and Gardner (1941)

Davis, Allison, Burleigh B. Gardner, Mary R. Gardner. 1941. *Deep South: A Social Anthropological Study of Caste and Class*. Chicago: The University of Chicago Press.

Goal:

- Examine relation between social class and informal interaction

Data Collection:

- Spent 9 months in Natchez, Mississippi
- Observed 18 women during 14 informal social events ("a day's work behind the counter of a store, a meeting of a women's club, a church supper, a card party, a supper party, a meeting of the PTA, etc")
- Recorded participation using "interviews, the records of participant observers, guest lists, and the newspapers"

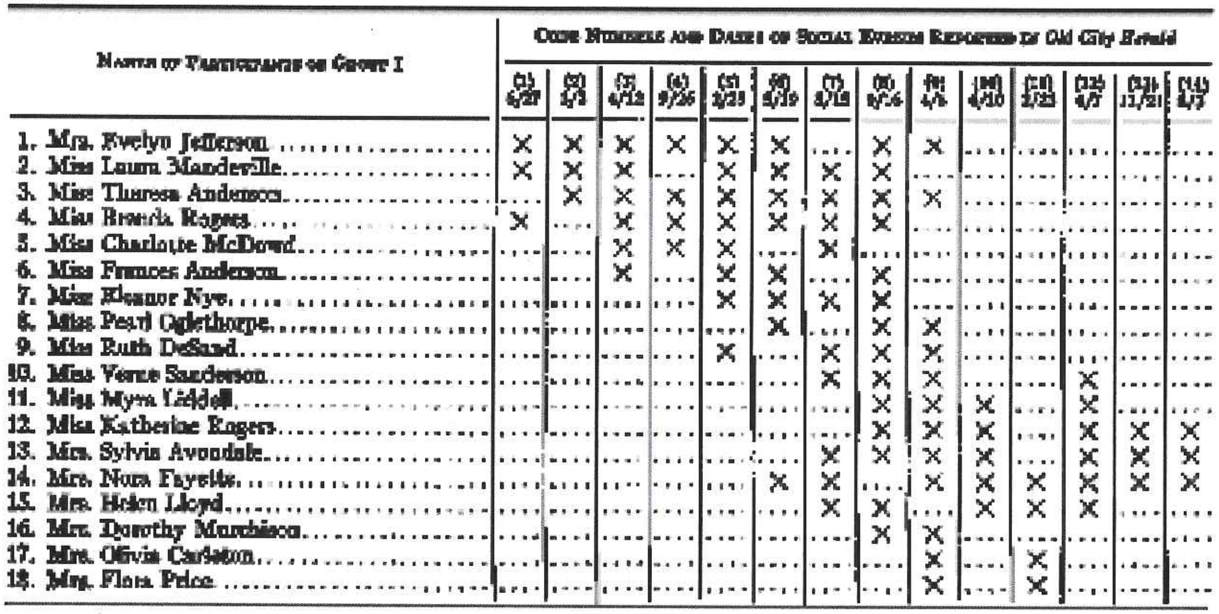
Southern Women

Research Questions

- Is the network of Southern women connected through social events?
- Do distinct social groups exist among these Southern women?
- Which of the women are more highly connected than others?

Southern Women Data

Davis, Gardner, and Gardner (1941)



Names of Participants of Group I	Code Numbers and Dates of Social Events Reported in *Old City Herald*													
	(1) 6/27	(2) 3/2	(3) 4/12	(4) 9/26	(5) 2/25	(6) 5/19	(7) 3/15	(8) 9/16	(9) 4/8	(10) 6/10	(11) 2/23	(12) 4/7	(13) 11/21	(14) 8/3
1. Mrs. Evelyn Jefferson	X	X	X	X	X	X		X	X					
2. Miss Laura Mandeville	X	X	X		X	X	X	X						
3. Miss Theresa Anderson		X	X	X	X	X	X	X	X					
4. Miss Brenda Rogers	X		X	X	X	X	X	X						
5. Miss Charlotte McDowd			X	X	X		X							
6. Miss Frances Anderson			X		X	X		X						
7. Miss Eleanor Nye					X	X	X	X						
8. Miss Pearl Oglethorpe						X		X	X					
9. Miss Ruth DeSand					X		X	X	X					
10. Miss Verne Sanderson							X	X	X			X		
11. Miss Myra Liddell								X	X	X		X		
12. Miss Katherine Rogers								X	X	X		X	X	X
13. Mrs. Sylvia Avondale							X	X	X	X		X	X	X
14. Mrs. Nora Fayette						X	X		X	X	X	X	X	X
15. Mrs. Helen Lloyd							X	X		X	X	X		
16. Mrs. Dorothy Murchison								X	X					
17. Mrs. Olivia Carleton									X		X			
18. Mrs. Flora Price									X		X			

Southern Women

Woman by Event Matrix

$$S = \begin{array}{r|cccccccccccccc|}
 & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} \\
\text{Eleanor} & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{Brenda} & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
\text{Dorothy} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{Verne} & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{Flora} & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Olivia} & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Laura} & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
\text{Evelyn} & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
\text{Pearl} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{Ruth} & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{Sylvia} & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
\text{Katherine} & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
\text{Myra} & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
\text{Theresa} & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
\text{Charlotte} & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{Frances} & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{Helen} & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
\text{Nora} & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1
\end{array}$$

Southern Women

Weighted Adjacency Matrix - Woman by Woman

$$A'_{ij} = SS^T = \begin{pmatrix}
4 & 4 & 1 & 2 & 0 & 0 & 4 & 3 & 2 & 3 & 2 & 1 & 1 & 4 & 2 & 3 & 2 & 2 \\
4 & 7 & 1 & 2 & 0 & 0 & 6 & 6 & 2 & 3 & 2 & 1 & 1 & 6 & 4 & 4 & 2 & 2 \\
1 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 \\
2 & 2 & 2 & 4 & 1 & 1 & 2 & 2 & 2 & 3 & 4 & 3 & 3 & 3 & 1 & 1 & 3 & 3 \\
0 & 0 & 1 & 1 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 2 \\
0 & 0 & 1 & 1 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 2 \\
4 & 6 & 1 & 2 & 0 & 0 & 7 & 6 & 2 & 3 & 2 & 1 & 1 & 6 & 3 & 4 & 2 & 2 \\
3 & 6 & 2 & 2 & 1 & 1 & 6 & 8 & 3 & 3 & 2 & 2 & 2 & 7 & 3 & 4 & 1 & 2 \\
2 & 2 & 2 & 2 & 1 & 1 & 2 & 3 & 3 & 2 & 2 & 2 & 2 & 3 & 0 & 2 & 1 & 2 \\
3 & 3 & 2 & 3 & 1 & 1 & 3 & 3 & 2 & 4 & 3 & 2 & 2 & 4 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 4 & 1 & 1 & 2 & 2 & 2 & 3 & 7 & 6 & 4 & 3 & 1 & 1 & 4 & 6 \\
1 & 1 & 2 & 3 & 1 & 1 & 1 & 2 & 2 & 2 & 6 & 6 & 4 & 2 & 0 & 1 & 3 & 5 \\
1 & 1 & 2 & 3 & 1 & 1 & 1 & 2 & 2 & 2 & 4 & 4 & 4 & 2 & 0 & 1 & 3 & 3 \\
4 & 6 & 2 & 3 & 1 & 1 & 6 & 7 & 3 & 4 & 3 & 2 & 2 & 8 & 4 & 4 & 2 & 3 \\
2 & 4 & 0 & 1 & 0 & 0 & 3 & 3 & 0 & 2 & 1 & 0 & 0 & 4 & 4 & 2 & 1 & 1 \\
3 & 4 & 1 & 1 & 0 & 0 & 4 & 4 & 2 & 2 & 1 & 1 & 1 & 4 & 2 & 4 & 1 & 1 \\
2 & 2 & 1 & 3 & 1 & 1 & 2 & 1 & 1 & 2 & 4 & 3 & 3 & 2 & 1 & 1 & 5 & 4 \\
2 & 2 & 1 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 6 & 5 & 3 & 3 & 1 & 1 & 4 & 8
\end{pmatrix}$$

$'_{ij}$ = # of events at which woman i and woman j jointly present if $i \neq j$.

$'_{ij}$ = total # of events attended by woman i if $i = j$.

Southern Women

Transition Matrix

Let $w_i = \sum_{j \neq i} A'_{ij}$,

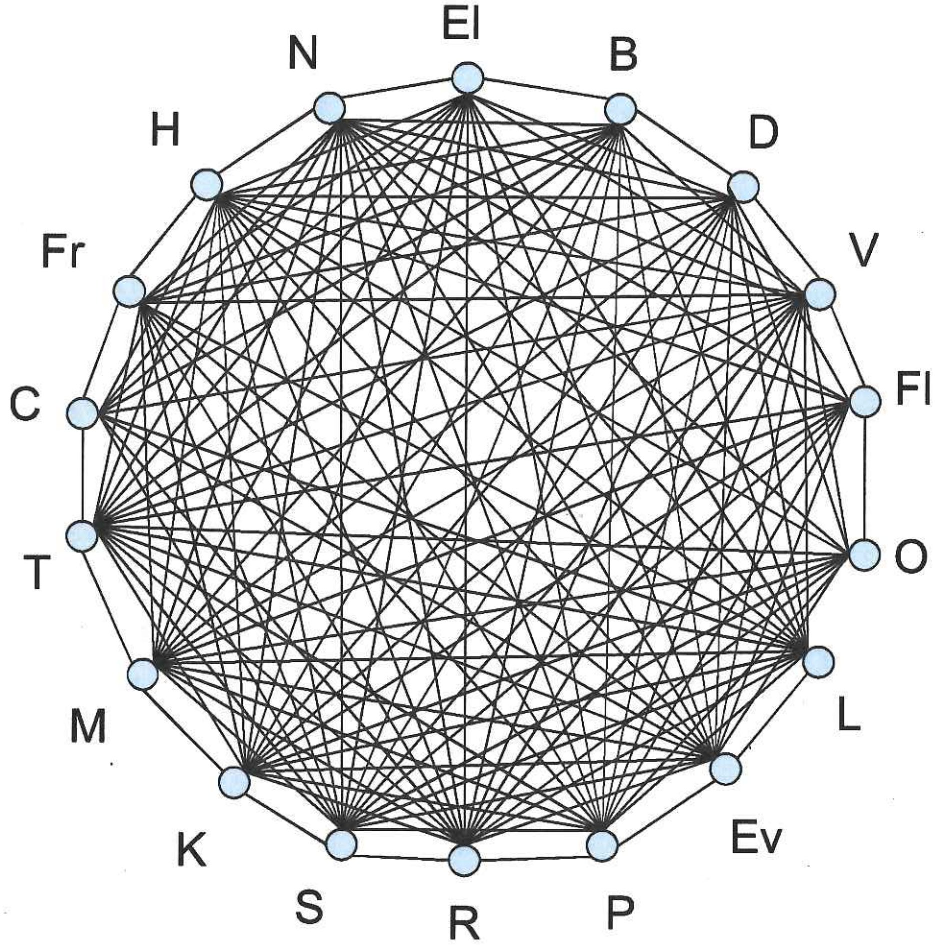
$$p_{ij} = \begin{cases} \dfrac{A'_{ij}}{w_i}, & i \neq j \\ 0, & i = j \end{cases}$$

$$P_{ij} = \begin{pmatrix}
0 & \frac{1}{9} & \frac{1}{36} & \frac{1}{18} & 0 & 0 & \frac{1}{9} & \frac{1}{12} & \frac{1}{18} & \frac{1}{12} & \frac{1}{18} & \frac{1}{36} & \frac{1}{36} & \frac{1}{9} & \frac{1}{18} & \frac{1}{12} & \frac{1}{18} & \frac{1}{18} \\
\frac{2}{23} & 0 & \frac{1}{46} & \frac{1}{23} & 0 & 0 & \frac{3}{23} & \frac{3}{23} & \frac{1}{23} & \frac{3}{46} & \frac{1}{23} & \frac{1}{46} & \frac{1}{46} & \frac{3}{23} & \frac{2}{23} & \frac{2}{23} & \frac{1}{23} & \frac{1}{23} \\
\frac{1}{24} & \frac{1}{24} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{1}{24} & \frac{1}{24} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{24} & \frac{1}{24} & \frac{1}{24} \\
\frac{1}{19} & \frac{1}{19} & \frac{1}{19} & 0 & \frac{1}{38} & \frac{1}{38} & \frac{1}{19} & \frac{1}{19} & \frac{1}{19} & \frac{3}{38} & \frac{2}{19} & \frac{3}{38} & \frac{3}{38} & \frac{3}{38} & \frac{1}{38} & \frac{1}{38} & \frac{3}{38} & \frac{3}{38} \\
0 & 0 & \frac{1}{14} & \frac{1}{14} & 0 & \frac{1}{7} & 0 & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & 0 & 0 & \frac{1}{14} & \frac{1}{7} \\
0 & 0 & \frac{1}{14} & \frac{1}{14} & \frac{1}{7} & 0 & 0 & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & \frac{1}{14} & 0 & 0 & \frac{1}{14} & \frac{1}{7} \\
\frac{4}{45} & \frac{2}{15} & \frac{1}{45} & \frac{2}{45} & 0 & 0 & 0 & \frac{2}{15} & \frac{2}{45} & \frac{1}{15} & \frac{2}{45} & \frac{1}{45} & \frac{1}{45} & \frac{2}{15} & \frac{1}{15} & \frac{4}{45} & \frac{2}{45} & \frac{2}{45} \\
\frac{3}{50} & \frac{3}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{50} & \frac{1}{50} & \frac{3}{25} & 0 & \frac{3}{50} & \frac{3}{50} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{7}{50} & \frac{3}{50} & \frac{2}{25} & \frac{1}{50} & \frac{1}{25} \\
\frac{2}{31} & \frac{2}{31} & \frac{2}{31} & \frac{2}{31} & \frac{1}{31} & \frac{1}{31} & \frac{2}{31} & \frac{3}{31} & 0 & \frac{2}{31} & \frac{2}{31} & \frac{2}{31} & \frac{2}{31} & \frac{3}{31} & 0 & \frac{2}{31} & \frac{1}{31} & \frac{2}{31} \\
\frac{3}{40} & \frac{3}{40} & \frac{1}{20} & \frac{3}{40} & \frac{1}{40} & \frac{1}{40} & \frac{3}{40} & \frac{3}{40} & \frac{1}{20} & 0 & \frac{3}{40} & \frac{1}{20} & \frac{1}{20} & \frac{1}{10} & \frac{1}{20} & \frac{1}{20} & \frac{1}{20} & \frac{1}{20} \\
\frac{1}{23} & \frac{1}{23} & \frac{1}{23} & \frac{2}{23} & \frac{1}{46} & \frac{1}{46} & \frac{1}{23} & \frac{1}{23} & \frac{1}{23} & \frac{3}{46} & 0 & \frac{3}{23} & \frac{2}{23} & \frac{3}{46} & \frac{1}{46} & \frac{1}{46} & \frac{2}{23} & \frac{3}{23} \\
\frac{1}{37} & \frac{1}{37} & \frac{2}{37} & \frac{3}{37} & \frac{1}{37} & \frac{1}{37} & \frac{1}{37} & \frac{2}{37} & \frac{2}{37} & \frac{2}{37} & \frac{6}{37} & 0 & \frac{4}{37} & \frac{2}{37} & 0 & \frac{1}{37} & \frac{3}{37} & \frac{5}{37} \\
\frac{1}{33} & \frac{1}{33} & \frac{2}{33} & \frac{1}{11} & \frac{1}{33} & \frac{1}{33} & \frac{1}{33} & \frac{2}{33} & \frac{2}{33} & \frac{2}{33} & \frac{4}{33} & \frac{4}{33} & 0 & \frac{2}{33} & 0 & \frac{1}{33} & \frac{1}{11} & \frac{1}{11} \\
\frac{4}{57} & \frac{2}{19} & \frac{2}{57} & \frac{1}{19} & \frac{1}{57} & \frac{1}{57} & \frac{2}{19} & \frac{7}{57} & \frac{1}{19} & \frac{4}{57} & \frac{1}{19} & \frac{2}{57} & \frac{2}{57} & 0 & \frac{4}{57} & \frac{4}{57} & \frac{2}{57} & \frac{1}{19} \\
\frac{1}{12} & \frac{1}{6} & 0 & \frac{1}{24} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{12} & \frac{1}{24} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{12} & \frac{1}{24} & \frac{1}{24} \\
\frac{3}{32} & \frac{1}{8} & \frac{1}{32} & \frac{1}{32} & 0 & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} & \frac{1}{16} & \frac{1}{32} & \frac{1}{32} & \frac{1}{32} & \frac{1}{8} & \frac{1}{16} & 0 & \frac{1}{32} & \frac{1}{32} \\
\frac{1}{17} & \frac{1}{17} & \frac{1}{34} & \frac{3}{34} & \frac{1}{34} & \frac{1}{34} & \frac{1}{17} & \frac{1}{34} & \frac{1}{34} & \frac{1}{17} & \frac{2}{17} & \frac{3}{34} & \frac{3}{34} & \frac{1}{17} & \frac{1}{34} & \frac{1}{34} & 0 & \frac{2}{17} \\
\frac{2}{43} & \frac{2}{43} & \frac{1}{43} & \frac{3}{43} & \frac{2}{43} & \frac{2}{43} & \frac{2}{43} & \frac{2}{43} & \frac{2}{43} & \frac{2}{43} & \frac{6}{43} & \frac{5}{43} & \frac{3}{43} & \frac{3}{43} & \frac{1}{43} & \frac{1}{43} & \frac{4}{43} & 0
\end{pmatrix}$$

P_{ij} = the influence of the current state of woman j on the state of woman i in the next time period.

Southern Women

Stationary Distribution



$$\pi P = \pi$$

Southern Women

Connectedness Ranking

$$\pi = \left({}^{1}\!/_{18},\ {}^{1}\!/_{14},\ {}^{1}\!/_{27},\ {}^{1}\!/_{17},\ {}^{1}\!/_{46},\ {}^{1}\!/_{46},\ {}^{3}\!/_{43},\ {}^{1}\!/_{13},\ {}^{4}\!/_{83},\ {}^{1}\!/_{16},\ {}^{1}\!/_{14},\ {}^{5}\!/_{87},\ {}^{2}\!/_{39},\ {}^{7}\!/_{79},\ {}^{1}\!/_{27},\ {}^{1}\!/_{20},\ {}^{1}\!/_{19},\ {}^{1}\!/_{15} \right)$$

Woman (i)	**Weight (π_i)**
Flora	1/46 = 0.0217
Olivia	1/46 = 0.0217
Dorothy	1/27 = 0.0373
Charlotte	1/27 = 0.0373
Pearl	4/83 = 0.0481
Frances	1/20 = 0.0497
Myra	2/39 = 0.0512
Helen	1/19 = 0.0528
Eleanor	1/18 = 0.0559
Katherine	5/87 = 0.0575
Verne	1/17 = 0.0590
Ruth	1/16 = 0.0621
Nora	1/15 = 0.0668
Laura	3/43 = 0.0699
Brenda	1/14 = 0.0714
Sylvia	1/14 = 0.0714
Evelyn	1/13 = 0.0776
Theresa	7/79 = 0.0885

Southern Women

Weighted Adjacency Matrix - Event by Event

$$B_{ij}' = S^T S = \begin{pmatrix}
4 & 0 & 0 & 2 & 2 & 3 & 0 & 1 & 2 & 0 & 1 & 1 & 0 & 1 \\
0 & 8 & 3 & 6 & 0 & 3 & 6 & 6 & 0 & 3 & 0 & 7 & 4 & 0 \\
0 & 3 & 3 & 2 & 0 & 2 & 3 & 3 & 0 & 2 & 0 & 3 & 2 & 0 \\
2 & 6 & 2 & 10 & 4 & 5 & 4 & 5 & 3 & 2 & 2 & 8 & 3 & 2 \\
2 & 0 & 0 & 4 & 6 & 5 & 0 & 1 & 5 & 0 & 3 & 5 & 0 & 3 \\
3 & 3 & 2 & 5 & 5 & 12 & 2 & 4 & 4 & 1 & 3 & 9 & 2 & 3 \\
0 & 6 & 3 & 4 & 0 & 2 & 6 & 5 & 0 & 3 & 0 & 5 & 4 & 0 \\
1 & 6 & 3 & 5 & 1 & 4 & 5 & 8 & 1 & 3 & 1 & 7 & 3 & 1 \\
2 & 0 & 0 & 3 & 5 & 4 & 0 & 1 & 5 & 0 & 3 & 4 & 0 & 3 \\
0 & 3 & 2 & 2 & 0 & 1 & 3 & 3 & 0 & 3 & 0 & 3 & 2 & 0 \\
1 & 0 & 0 & 2 & 3 & 3 & 0 & 1 & 3 & 0 & 3 & 2 & 0 & 3 \\
1 & 7 & 3 & 8 & 5 & 9 & 5 & 7 & 4 & 3 & 2 & 14 & 3 & 2 \\
0 & 4 & 2 & 3 & 0 & 2 & 4 & 3 & 0 & 2 & 0 & 3 & 4 & 0 \\
1 & 0 & 0 & 2 & 3 & 3 & 0 & 1 & 3 & 0 & 3 & 2 & 0 & 3
\end{pmatrix}$$

$''_{ij}$ = # of women who participated in both events i and j if $i \neq j$.

$''_{ij}$ = total # of women who attended event i if $i = j$.

Southern Women

Weighted Adjacency Matrix - Event by Event

$$B_{ij}' = S^T S = \begin{array}{c} \begin{array}{cccccccccccccc} & & & 4 & & 6 & & 8 & & & & 12 & & \end{array} \\ \left(\begin{array}{cccccccccccccc} 4 & 0 & 0 & 2 & 2 & 3 & 0 & 1 & 2 & 0 & 1 & 1 & 0 & 1 \\ 0 & 8 & 3 & 6 & 0 & 3 & 6 & 6 & 0 & 3 & 0 & 7 & 4 & 0 \\ 0 & 3 & 3 & 2 & 0 & 2 & 3 & 3 & 0 & 2 & 0 & 3 & 2 & 0 \\ 2 & 6 & 2 & 10 & 4 & 5 & 4 & 5 & 3 & 2 & 2 & 8 & 3 & 2 \\ 2 & 0 & 0 & 4 & 6 & 5 & 0 & 1 & 5 & 0 & 3 & 5 & 0 & 3 \\ 3 & 3 & 2 & 5 & 5 & 12 & 2 & 4 & 4 & 1 & 3 & 9 & 2 & 3 \\ 0 & 6 & 3 & 4 & 0 & 2 & 6 & 5 & 0 & 3 & 0 & 5 & 4 & 0 \\ 1 & 6 & 3 & 5 & 1 & 4 & 5 & 8 & 1 & 3 & 1 & 7 & 3 & 1 \\ 2 & 0 & 0 & 3 & 5 & 4 & 0 & 1 & 5 & 0 & 3 & 4 & 0 & 3 \\ 0 & 3 & 2 & 2 & 0 & 1 & 3 & 3 & 0 & 3 & 0 & 3 & 2 & 0 \\ 1 & 0 & 0 & 2 & 3 & 3 & 0 & 1 & 3 & 0 & 3 & 2 & 0 & 3 \\ 1 & 7 & 3 & 8 & 5 & 9 & 5 & 7 & 4 & 3 & 2 & 14 & 3 & 2 \\ 0 & 4 & 2 & 3 & 0 & 2 & 4 & 3 & 0 & 2 & 0 & 3 & 4 & 0 \\ 1 & 0 & 0 & 2 & 3 & 3 & 0 & 1 & 3 & 0 & 3 & 2 & 0 & 3 \end{array}\right) \end{array}$$

Remove events that bring women together

$''_{ij}$ = # of women who participated in both events i and j if $i \neq j$.

$''_{ij}$ = total # of women who attended event i if $i = j$.

Breiger, Ronald L. 1974. "The Duality of Persons and Groups." *Social Forces*, Vol. 53, No. 2, Special Issue, pp. 181-190.

Southern Women

Modified Woman by Event Matrix

$$S' = \begin{array}{l|cccccccccc|}
 & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{5} & \mathbf{7} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{13} & \mathbf{14} \\
\text{Eleanor} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Brenda} & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\text{Dorothy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Verne} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Flora} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Olivia} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Laura} & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
\text{Evelyn} & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
\text{Pearl} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Ruth} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{Sylvia} & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
\text{Katherine} & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
\text{Myra} & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
\text{Theresa} & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
\text{Charlotte} & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
\text{Frances} & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\text{Helen} & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
\text{Nora} & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}$$

Isolated Nodes

Southern Women

Weighted Adjacency Matrix - Modified Woman by Woman

(Dorothy and Pearl Removed)

$$M_{ij}' = S'S'^{T} = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 4 & 0 & 0 & 0 & 3 & 4 & 1 & 0 & 0 & 0 & 3 & 3 & 2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 3 & 0 & 0 & 0 & 4 & 4 & 1 & 0 & 0 & 0 & 3 & 2 & 2 & 0 & 0 \\
1 & 4 & 0 & 0 & 0 & 4 & 5 & 1 & 0 & 0 & 0 & 4 & 3 & 2 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 4 & 4 & 2 & 0 & 0 & 0 & 2 & 4 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 4 & 4 & 2 & 0 & 0 & 0 & 2 & 4 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\
1 & 3 & 0 & 0 & 0 & 3 & 4 & 1 & 0 & 0 & 0 & 4 & 3 & 2 & 0 & 0 \\
1 & 3 & 0 & 0 & 0 & 2 & 3 & 1 & 0 & 0 & 0 & 3 & 3 & 2 & 0 & 0 \\
1 & 2 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 3 & 3 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 4 & 4 & 2 & 0 & 0 & 0 & 3 & 5
\end{pmatrix}$$

$''_{ij}$ = # of events at which
woman *i* and woman *j*
jointly present if $i \neq j$.

$''_{ij}$ = total # of events
attended by woman *i*
if $i = j$.

Southern Women

Modified Transition Matrix

Let $w'_i = \sum_{j \neq i} M'_{ij}$,

$$q_{ij} = \begin{cases} \dfrac{M'_{ij}}{w'_i}, & i \neq j \\ 0, & i = j \end{cases}$$

Q_{ij} = the influence of the current state of woman j on the state of woman i in the next time period.

$$Q_{ij} = \begin{pmatrix}
0 & 1/7 & 0 & 0 & 0 & 1/7 & 1/7 & 1/7 & 0 & 0 & 0 & 1/7 & 1/7 & 1/7 & 0 & 0 \\
1/17 & 0 & 0 & 0 & 0 & 3/17 & 4/17 & 1/17 & 0 & 0 & 0 & 3/17 & 3/17 & 2/17 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/5 & 1/5 & 1/5 & 0 & 0 & 0 & 1/5 & 1/5 \\
0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 \\
0 & 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 \\
1/16 & 3/16 & 0 & 0 & 0 & 0 & 1/4 & 1/16 & 0 & 0 & 0 & 3/16 & 1/8 & 1/8 & 0 & 0 \\
1/19 & 4/19 & 0 & 0 & 0 & 4/19 & 0 & 1/19 & 0 & 0 & 0 & 4/19 & 3/19 & 2/19 & 0 & 0 \\
1/7 & 1/7 & 0 & 0 & 0 & 1/7 & 1/7 & 0 & 0 & 0 & 0 & 1/7 & 1/7 & 1/7 & 0 & 0 \\
0 & 0 & 1/13 & 0 & 0 & 0 & 0 & 0 & 0 & 4/13 & 2/13 & 0 & 0 & 0 & 2/13 & 4/13 \\
0 & 0 & 1/13 & 0 & 0 & 0 & 0 & 0 & 4/13 & 0 & 2/13 & 0 & 0 & 0 & 2/13 & 4/13 \\
0 & 0 & 1/9 & 0 & 0 & 0 & 0 & 0 & 2/9 & 2/9 & 0 & 0 & 0 & 0 & 2/9 & 2/9 \\
1/17 & 3/17 & 0 & 0 & 0 & 3/17 & 4/17 & 1/17 & 0 & 0 & 0 & 0 & 3/17 & 2/17 & 0 & 0 \\
1/15 & 1/5 & 0 & 0 & 0 & 2/15 & 1/5 & 1/15 & 0 & 0 & 0 & 1/5 & 0 & 2/15 & 0 & 0 \\
1/12 & 1/6 & 0 & 0 & 0 & 1/6 & 1/6 & 1/12 & 0 & 0 & 0 & 1/6 & 1/6 & 0 & 0 & 0 \\
0 & 0 & 1/12 & 1/12 & 1/12 & 0 & 0 & 0 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & 0 & 1/4 \\
0 & 0 & 1/16 & 1/16 & 1/16 & 0 & 0 & 0 & 1/4 & 1/4 & 1/8 & 0 & 0 & 0 & 3/16 & 0
\end{pmatrix}$$

Southern Women

Modified Transition Matrix

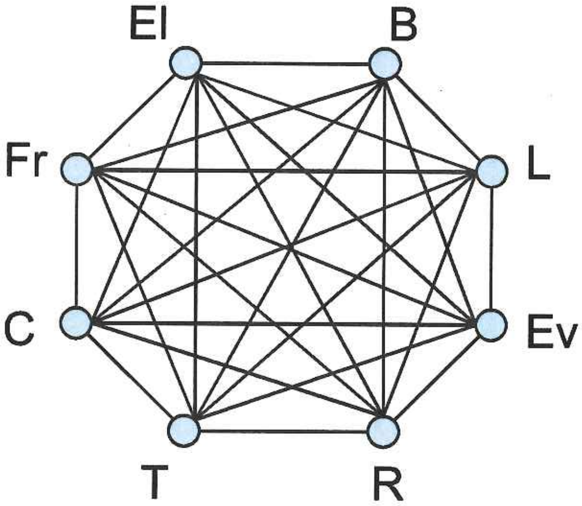
$$Q_{ij} = \left(\begin{array}{cccccccc|cccccccc}
0 & \tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{17} & 0 & \tfrac{3}{17} & \tfrac{4}{17} & \tfrac{1}{17} & \tfrac{3}{17} & \tfrac{3}{17} & \tfrac{2}{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{16} & \tfrac{3}{16} & 0 & \tfrac{1}{4} & \tfrac{1}{16} & \tfrac{3}{16} & \tfrac{1}{8} & \tfrac{1}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{19} & \tfrac{4}{19} & \tfrac{4}{19} & 0 & \tfrac{1}{19} & \tfrac{4}{19} & \tfrac{3}{19} & \tfrac{2}{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & 0 & \tfrac{1}{7} & \tfrac{1}{7} & \tfrac{1}{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{17} & \tfrac{3}{17} & \tfrac{3}{17} & \tfrac{4}{17} & \tfrac{1}{17} & 0 & \tfrac{3}{17} & \tfrac{2}{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{15} & \tfrac{1}{5} & \tfrac{2}{15} & \tfrac{1}{5} & \tfrac{1}{15} & \tfrac{1}{5} & 0 & \tfrac{2}{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tfrac{1}{12} & \tfrac{1}{12} & \tfrac{1}{12} & \tfrac{1}{6} & \tfrac{1}{6} & \tfrac{1}{6} & \tfrac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{5} & \tfrac{1}{5} & \tfrac{1}{5} & \tfrac{1}{5} & \tfrac{1}{5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{3} & 0 & 0 & 0 & \tfrac{1}{3} & \tfrac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{3} & 0 & 0 & 0 & 0 & \tfrac{1}{3} & \tfrac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{13} & 0 & 0 & 0 & \tfrac{4}{13} & \tfrac{2}{13} & \tfrac{2}{13} & \tfrac{4}{13} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{13} & 0 & 0 & \tfrac{4}{13} & 0 & \tfrac{2}{13} & \tfrac{2}{13} & \tfrac{4}{13} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{9} & 0 & 0 & \tfrac{2}{9} & \tfrac{2}{9} & 0 & \tfrac{2}{9} & \tfrac{2}{9} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{12} & \tfrac{1}{12} & \tfrac{1}{12} & \tfrac{1}{6} & \tfrac{1}{6} & \tfrac{1}{6} & 0 & \tfrac{1}{4} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tfrac{1}{16} & \tfrac{1}{16} & \tfrac{1}{16} & \tfrac{1}{4} & \tfrac{1}{4} & \tfrac{1}{8} & \tfrac{3}{16} & 0
\end{array}\right)$$

Southern Women

Component 1

$$Q_1 = \begin{pmatrix} 0 & {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 \\ {}^1\!/_{17} & 0 & {}^3\!/_{17} & {}^4\!/_{17} & {}^1\!/_{17} & {}^3\!/_{17} & {}^3\!/_{17} & {}^2\!/_{17} \\ {}^1\!/_{16} & {}^3\!/_{16} & 0 & {}^1\!/_4 & {}^1\!/_{16} & {}^3\!/_{16} & {}^1\!/_8 & {}^1\!/_8 \\ {}^1\!/_{19} & {}^4\!/_{19} & {}^4\!/_{19} & 0 & {}^1\!/_{19} & {}^4\!/_{19} & {}^3\!/_{19} & {}^2\!/_{19} \\ {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 & 0 & {}^1\!/_7 & {}^1\!/_7 & {}^1\!/_7 \\ {}^1\!/_{17} & {}^3\!/_{17} & {}^3\!/_{17} & {}^4\!/_{17} & {}^1\!/_{17} & 0 & {}^3\!/_{17} & {}^2\!/_{17} \\ {}^1\!/_{15} & {}^1\!/_5 & {}^2\!/_{15} & {}^1\!/_5 & {}^1\!/_{15} & {}^1\!/_5 & 0 & {}^2\!/_{15} \\ {}^1\!/_{12} & {}^1\!/_{12} & {}^1\!/_{12} & {}^1\!/_6 & {}^1\!/_6 & {}^1\!/_6 & {}^1\!/_4 & 0 \end{pmatrix}$$

$$\pi = \left({}^2\!/_{31},\ \ {}^{11}\!/_{75},\ \ {}^7\!/_{51},\ \ {}^{17}\!/_{99},\ \ {}^6\!/_{83},\ \ {}^2\!/_{13},\ \ {}^1\!/_7,\ \ {}^7\!/_{64}\right)$$



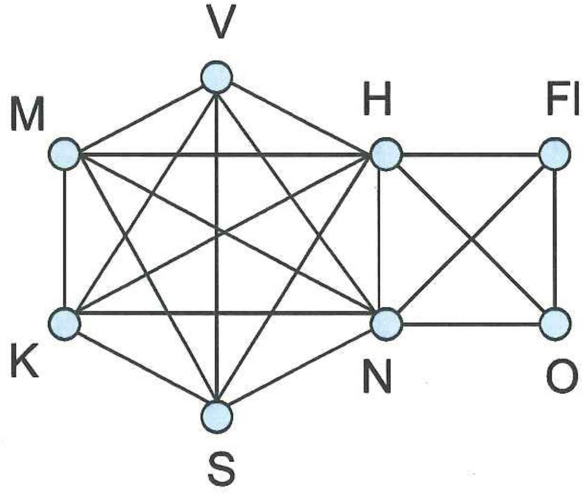
Woman (i)	**Weight (π_i)**
Eleanor	2/31 = 0.0644
Ruth	6/83 = 0.0723
Frances	7/64 = 0.1094
Laura	7/51 = 0.1371
Charlotte	1/7 = 0.1442
Brenda	11/75 = 0.1466
Theresa	2/13 = 0.1543
Evelyn	17/99 = 0.1717

Southern Women

Component 2

$$Q_2 = \begin{pmatrix} 0 & 0 & 0 & \tfrac{1}{5} & \tfrac{1}{5} & \tfrac{1}{5} & \tfrac{1}{5} & \tfrac{1}{5} \\ 0 & 0 & \tfrac{1}{3} & 0 & 0 & 0 & \tfrac{1}{3} & \tfrac{1}{3} \\ 0 & \tfrac{1}{3} & 0 & 0 & 0 & 0 & \tfrac{1}{3} & \tfrac{1}{3} \\ \tfrac{1}{13} & 0 & 0 & 0 & \tfrac{4}{13} & \tfrac{2}{13} & \tfrac{2}{13} & \tfrac{4}{13} \\ \tfrac{1}{13} & 0 & 0 & \tfrac{4}{13} & 0 & \tfrac{2}{13} & \tfrac{2}{13} & \tfrac{4}{13} \\ \tfrac{1}{9} & 0 & 0 & \tfrac{2}{9} & \tfrac{2}{9} & 0 & \tfrac{2}{9} & \tfrac{2}{9} \\ \tfrac{1}{12} & \tfrac{1}{12} & \tfrac{1}{12} & \tfrac{1}{6} & \tfrac{1}{6} & \tfrac{1}{6} & 0 & \tfrac{1}{4} \\ \tfrac{1}{16} & \tfrac{1}{16} & \tfrac{1}{16} & \tfrac{1}{4} & \tfrac{1}{4} & \tfrac{1}{8} & \tfrac{3}{16} & 0 \end{pmatrix}$$

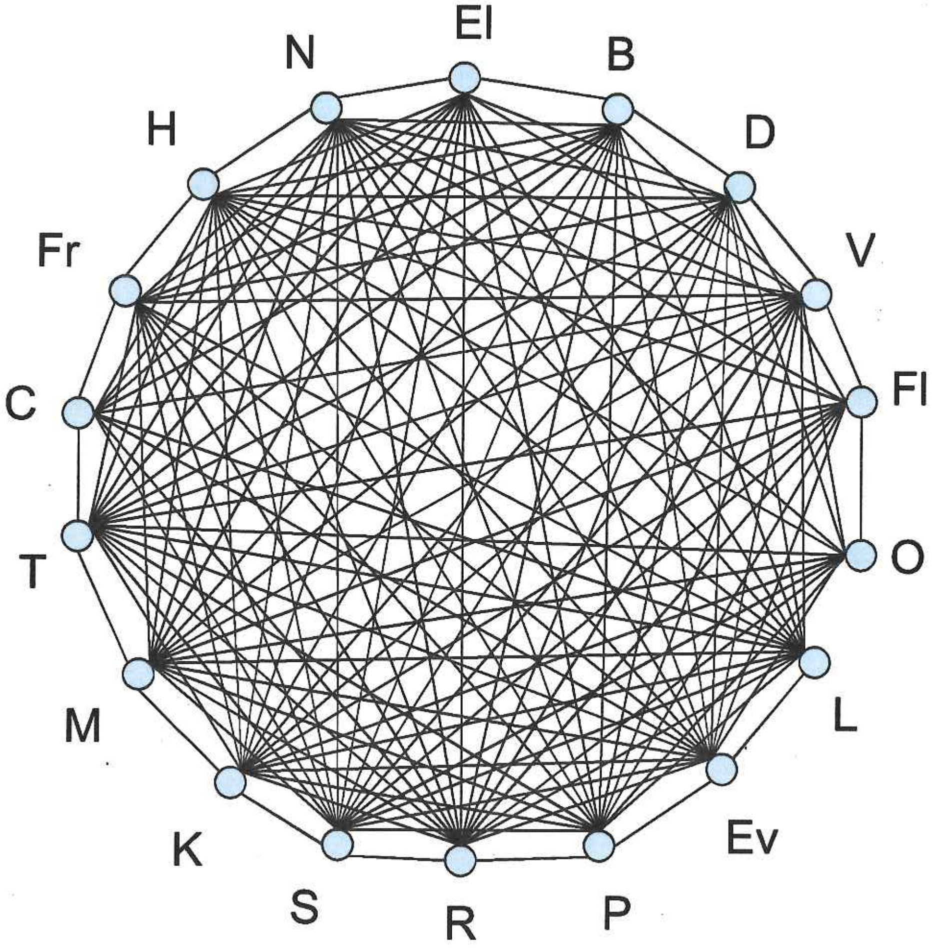
$$\pi = \left(\tfrac{5}{74},\ \tfrac{3}{74},\ \tfrac{3}{74},\ \tfrac{13}{74},\ \tfrac{13}{74},\ \tfrac{9}{74},\ \tfrac{6}{37},\ \tfrac{8}{37}\right)$$

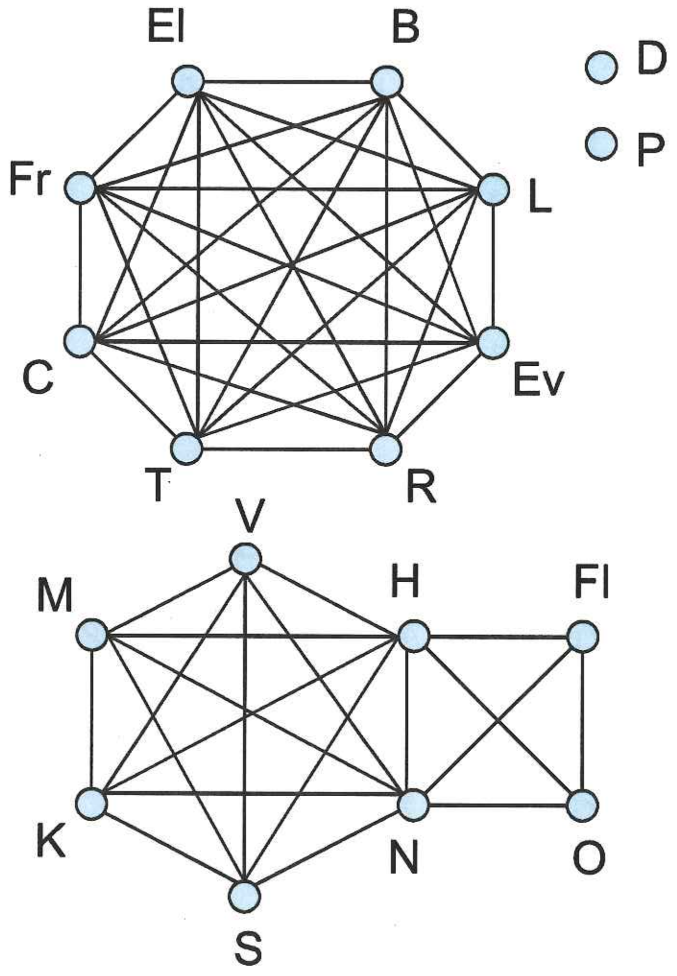


Woman (i)	**Weight (π_i)**
Flora	3/74 = 0.0405
Olivia	3/74 = 0.0405
Verne	5/74 = 0.0676
Myra	9/74 = 0.1216
Helen	6/37 = 0.1622
Sylvia	13/74 = 0.1757
Katherine	13/74 = 0.1757
Nora	8/37 = 0.2162

Southern Women

Connectedness vs. Isolated Components





Graph Partitioning

Spectral Partitioning Method

Laplacian matrix: Consider a graph with an adjacency matrix *A* containing the elements $A_{ij} = 1$ if there is an edge connecting vertices *i* and *j*, and $A_{ij} = 0$ otherwise. The *Laplacian matrix* of the graph is defined as

$$L_{ij} = \begin{cases} k_i, & i = j \\ -1, & \text{if } i \neq j \text{ and } i \sim j \\ 0, & \text{otherwise} \end{cases}$$

where k_i is the degree of vertex *i.*

The second-smallest eigenvalue of the Laplacian matrix is known as the *algebraic connectivity* of the graph. The corresponding eigenvector is known as the *Fiedler vector* and can be used to partition the vertices.

Graph Partitioning

Spectral Partitioning Method - eigenvalue and eigenvector

Definition: If A is an n x n matrix, then a nonzero vector $\mathbf{x}$ is called an eigenvector of A if $A\mathbf{x}$ is a scalar multiple of $\mathbf{x}$; that is, for some scalar λ, we have that $A\mathbf{x} = \lambda\mathbf{x}$. The scalar λ is called an *eigenvalue* of A, and $\mathbf{x}$ is called an *eigenvector* of A corresponding to λ.

Recall: The stationary distribution of a Markov chain is a left-hand eigenvector of the transition probability matrix T corresponding to eigenvalue 1.

$$\pi T = \pi$$

Graph Partitioning

Spectral Partitioning Method - Example

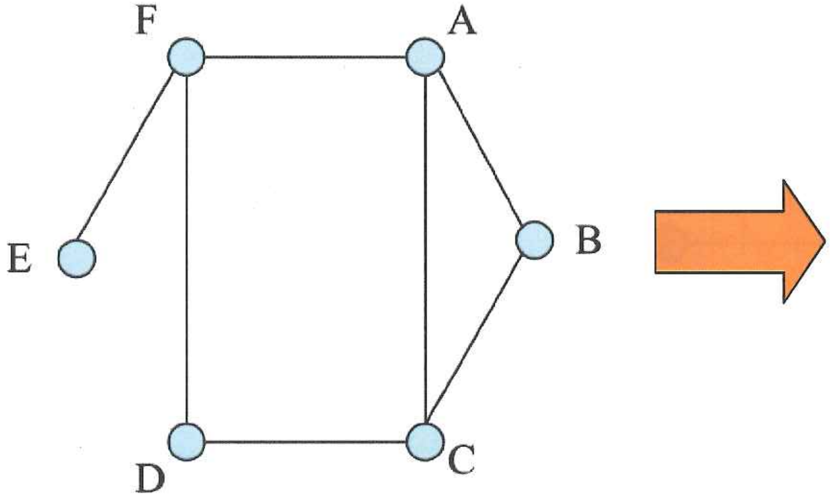


Figure 1. Toy Example – Graph *G*

$$L = \begin{pmatrix} 3 & -1 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -1 & -1 & 3 \end{pmatrix}$$

Graph Partitioning

Spectral Partitioning Method - Example

Algebraic Connectivity: $\lambda_2 = 0.7216$

Fiedler Vector:

$$\mathbf{x}_2 = (\, -0.2209, -0.4149, -0.3094, -0.0692, 0.7935, 0.2209)$$

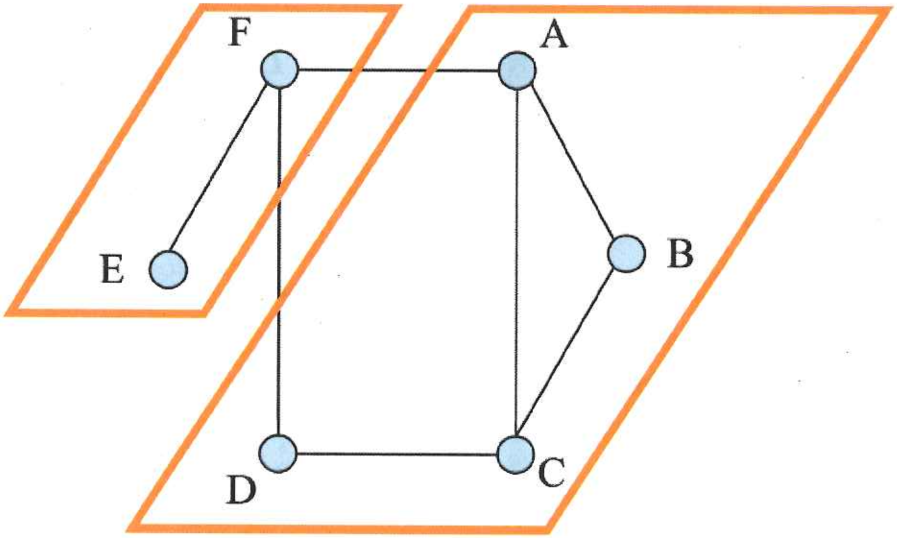


Figure 2. Partition of graph G found by eigenvectors of the Laplacian matrix

Graph Partitioning

Modularity Partitioning Method

Modularity matrix: Consider a graph with an adjacency matrix *A* containing the elements $A_{ij} = 1$ if there is an edge connecting vertices *i* and *j*, and $A_{ij} = 0$ otherwise, where *m* represents the number of edges in the graph. The *modularity matrix* of the graph is defined as

$$B_{ij} = A_{ij} - \frac{k_i k_j}{2m},$$

where k_i is the degree of vertex *i*, and k_i is the degree of vertex *j*.

In the case of the modularity matrix, the eigenvector corresponding to the largest eigenvalue can be used to partition the vertices.

Graph Partitioning

Modularity Partitioning Method - Example

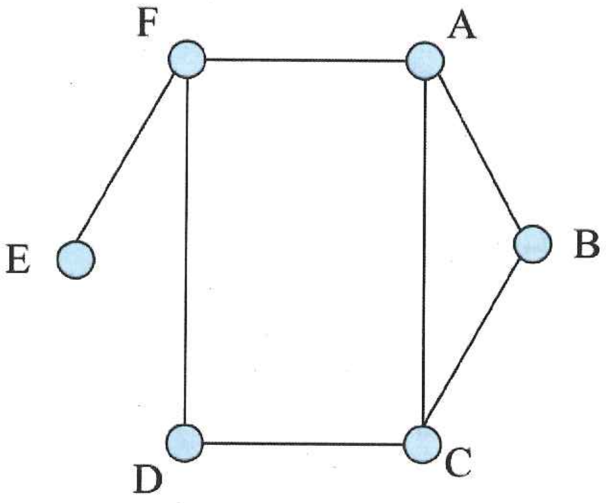


Figure 1. Toy Example – Graph *G*

$$B_{ij} = \begin{pmatrix} {}^{-9}/_{14} & {}^{4}/_{7} & {}^{5}/_{14} & {}^{-3}/_{7} & {}^{-3}/_{14} & {}^{5}/_{14} \\ {}^{4}/_{7} & {}^{-2}/_{7} & {}^{4}/_{7} & {}^{-2}/_{7} & {}^{-1}/_{7} & {}^{-3}/_{7} \\ {}^{5}/_{14} & {}^{4}/_{7} & {}^{-9}/_{14} & {}^{4}/_{7} & {}^{-3}/_{14} & {}^{-9}/_{14} \\ {}^{-3}/_{7} & {}^{-2}/_{7} & {}^{4}/_{7} & {}^{-2}/_{7} & {}^{-1}/_{7} & {}^{4}/_{7} \\ {}^{-3}/_{14} & {}^{-1}/_{7} & {}^{-3}/_{14} & {}^{-1}/_{7} & {}^{-1}/_{14} & {}^{11}/_{14} \\ {}^{5}/_{14} & {}^{-3}/_{7} & {}^{-9}/_{14} & {}^{4}/_{7} & {}^{11}/_{14} & {}^{-9}/_{14} \end{pmatrix}$$

Graph Partitioning

Modularity Partitioning Method - Example

Largest Eigenvalue: $\lambda_1 = 1.1061$

Corresponding Eigenvector:

$$\mathbf{x}_1 = (0.2478, 0.5063, 0.4045, -0.1682, -0.4922, -0.4982)$$

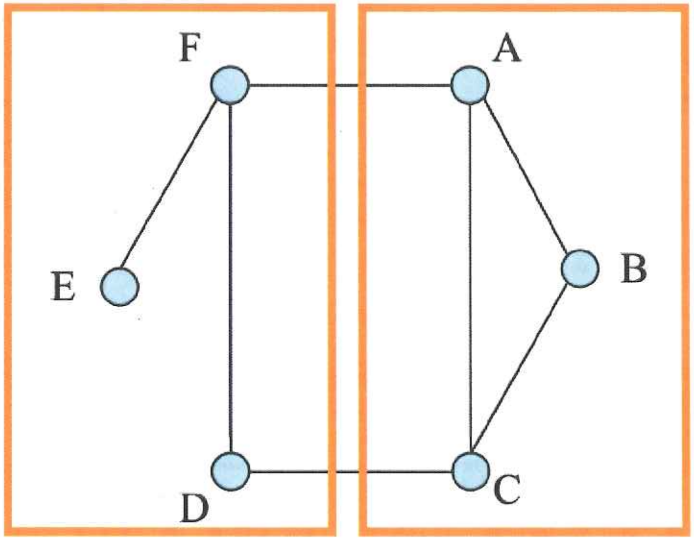


Figure 3. Partition of graph G found by eigenvectors of the Modularity matrix

Graph Partitioning

Modularity Partitioning Method - Southern Women

$$\begin{array}{l|cccccccc|cccccc|cc|cc}
\text{Brenda} & 0 & 4 & 4 & 6 & 4 & 6 & 3 & 6 & 2 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 0 & 0 \\
\text{Charlotte} & 4 & 0 & 2 & 3 & 2 & 3 & 2 & 4 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
\text{Eleanor} & 4 & 2 & 0 & 3 & 3 & 4 & 3 & 4 & 2 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 0 & 0 \\
\text{Evelyn} & 6 & 3 & 3 & 0 & 4 & 6 & 3 & 7 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 1 & 1 \\
\text{Frances} & 4 & 2 & 3 & 4 & 0 & 4 & 2 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 0 \\
\text{Laura} & 6 & 3 & 4 & 6 & 4 & 0 & 3 & 6 & 2 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 0 & 0 \\
\text{Ruth} & 3 & 2 & 3 & 3 & 2 & 3 & 0 & 4 & 2 & 2 & 2 & 2 & 3 & 3 & 2 & 2 & 1 & 1 \\
\text{Theresa} & 6 & 4 & 4 & 7 & 4 & 6 & 4 & 0 & 2 & 2 & 2 & 3 & 3 & 3 & 2 & 3 & 1 & 1 \\
\hline
\text{Helen} & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 2 & 0 & 3 & 3 & 4 & 4 & 3 & 1 & 1 & 1 & 1 \\
\text{Katherine} & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 2 & 3 & 0 & 4 & 5 & 6 & 3 & 2 & 2 & 1 & 1 \\
\text{Myra} & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 2 & 3 & 4 & 0 & 3 & 4 & 3 & 2 & 2 & 1 & 1 \\
\text{Nora} & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 3 & 4 & 5 & 3 & 0 & 6 & 3 & 1 & 2 & 2 & 2 \\
\text{Sylvia} & 2 & 1 & 2 & 2 & 1 & 2 & 3 & 3 & 4 & 6 & 4 & 6 & 0 & 4 & 2 & 2 & 1 & 1 \\
\text{Verne} & 2 & 1 & 2 & 2 & 1 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 4 & 0 & 2 & 2 & 1 & 1 \\
\hline
\text{Dorothy} & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 0 & 2 & 1 & 1 \\
\text{Pearl} & 2 & 0 & 2 & 3 & 2 & 2 & 2 & 3 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 \\
\hline
\text{Flora} & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 0 & 2 \\
\text{Olivia} & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 2 & 0
\end{array}$$

Southern Women

Comparison of Partitions

Brenda	0	4	4	6	4	6	3	6	2	1	1	2	2	2	1	2	0	0
Charlotte	4	0	2	3	2	3	2	4	1	0	0	1	1	1	0	0	0	0
Eleanor	4	2	0	3	3	4	3	4	2	1	1	2	2	2	1	2	0	0
Evelyn	6	3	3	0	4	6	3	7	1	2	2	2	2	2	2	3	1	1
Frances	4	2	3	4	0	4	2	4	1	1	1	1	1	1	1	2	0	0
Laura	6	3	4	6	4	0	3	6	2	1	1	2	2	2	1	2	0	0
Ruth	3	2	3	3	2	3	0	4	2	2	2	2	3	3	2	2	1	1
Theresa	6	4	4	7	4	6	4	0	2	2	2	3	3	3	2	3	1	1
Helen	2	1	2	1	1	2	2	2	0	3	3	4	4	3	1	1	1	1
Katherine	1	0	1	2	1	1	2	2	3	0	4	5	6	3	2	2	1	1
Myra	1	0	1	2	1	1	2	2	3	4	0	3	4	3	2	2	1	1
Nora	2	1	2	2	1	2	2	3	4	5	3	0	6	3	1	2	2	2
Sylvia	2	1	2	2	1	2	3	3	4	6	4	6	0	4	2	2	1	1
Verne	2	1	2	2	1	2	3	3	3	3	3	3	4	0	2	2	1	1
Dorothy	1	0	1	2	1	1	2	2	1	2	2	1	2	2	0	2	1	1
Pearl	2	0	2	3	2	2	2	3	1	2	2	2	2	2	2	0	1	1
Flora	0	0	0	1	0	0	1	1	1	1	1	2	1	1	1	1	0	2
Olivia	0	0	0	1	0	0	1	1	1	1	1	2	1	1	1	1	2	0